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Mathematical formulation of the Nigeria road network model of multiple sources to multiple destinations

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Abstract
In this research work, we examine Nigeria road network of multiple sources-to-multiple destinations from the coastal towns of Lagos, Warri/Asaba, Port Harcourt and Calabar through the intermediary routes to multiple destinations of the border town of Sokoto, Katsina and Maiduguri. The mathematical model was developed using positive edge weights based on the current road distances between state capitals obtained from Nigeria kilometer chart of state capitals. Lingo computer application package was used to solve the formulated problem consisting of 1 objective function, 75 constraints and 441 variables to identify the shortest routes from given multiple sources-to-multiple destinations. The shortest routes obtained compared favorably with the actual ones and were found to be the best.

Keywords: Road network, multiple sources, multiple destinations, mathematical formulation, shortest route, linear programming

Introduction

Everything is a network, whenever we look at the interactions between things, a network is formed implicitly. Network of roads have become very important for road transporters, since it serves as a channel of moving people, goods and services from one place to another. In recent times, we have also observed that a lot of traffic congestion happens on our roads as a result of more influx of vehicles and poor state of some of the roads. Anyanwu *et al.* (1997) [2] and Nwafor and Onya (2019) [8] noted that, of all commodity movements to and from the sea-ports and airport, at least, two-thirds are handled by road transport while up to 90% of all other internal movement of goods and persons take place by roads. Olaniyi and Oniru (2017) [10], also noted that, road transportation is demanded to execute the objectives of every other sectors in the economy. Not only does transportation provides mobility for people and goods, it helps shape an area's economic health and quality of life. Because of the high pertinence of road transportation, it is expedient for a country or nation to embark on integrating man, material, money and machinery towards the realization of diversified modes of transport before such country will boast to have achieved a diversified economy and a sustainable development. The Nigeria road network linked together all over the 36 states and the Federal Capital Territory with arterial roads which are interconnected with sub-arterial, distributor or collector roads and local roads are full of challenges, particularly with pavement, although, this research work did not consider the challenges, rather the mathematical model of shortest route was designed and developed for finding the optimal routes from coaster towns through the several intermediate cities to border towns.

Nigeria has a national road network of about 200,000km; of this total, federal roads make up 18 per cent (about 36,000km), state roads 15 per cent (about 30,000km) and local government roads 67 per cent (about 134,000km) as shown in Figure 2, with most local government roads being unpaved. The road sector accounts for about 90 per cent of all freight and passenger movements in the country. Although the federal road network constitutes 18 per cent of the total national network, it accounts for about 70 per cent of the national vehicular and freight traffic. As at 2012, an estimated 40 per cent of the federal road network was in poor condition (in need of rehabilitation); 30 per cent in fair condition requiring periodic maintenance); and 27 per cent in good condition (requiring only routine maintenance). The remaining 3 per cent consists of unpaved trunk roads that need to be paved. In the case of state roads, 78 per cent is in poor condition, with 87 per cent of local government roads also considered to be in poor condition (Source: National Planning Commission, 2015) [7].

It has been observed that other routes can be used for the same purpose of transporting people, goods and services from one location to the other, some of these roads can take a very long time which results in delay and even breakdown of vehicle, thus increasing the cost of maintaining such vehicles on the path of management. As a result, this research work seeks to model, formulate and obtain the shortest possible route for vehicular movement from multiple sources to multiple destinations through multiple-roads network system so as to minimize the cost of fueling, maintenance and loss of customer's lives through accidents and timely arrival to their destinations.

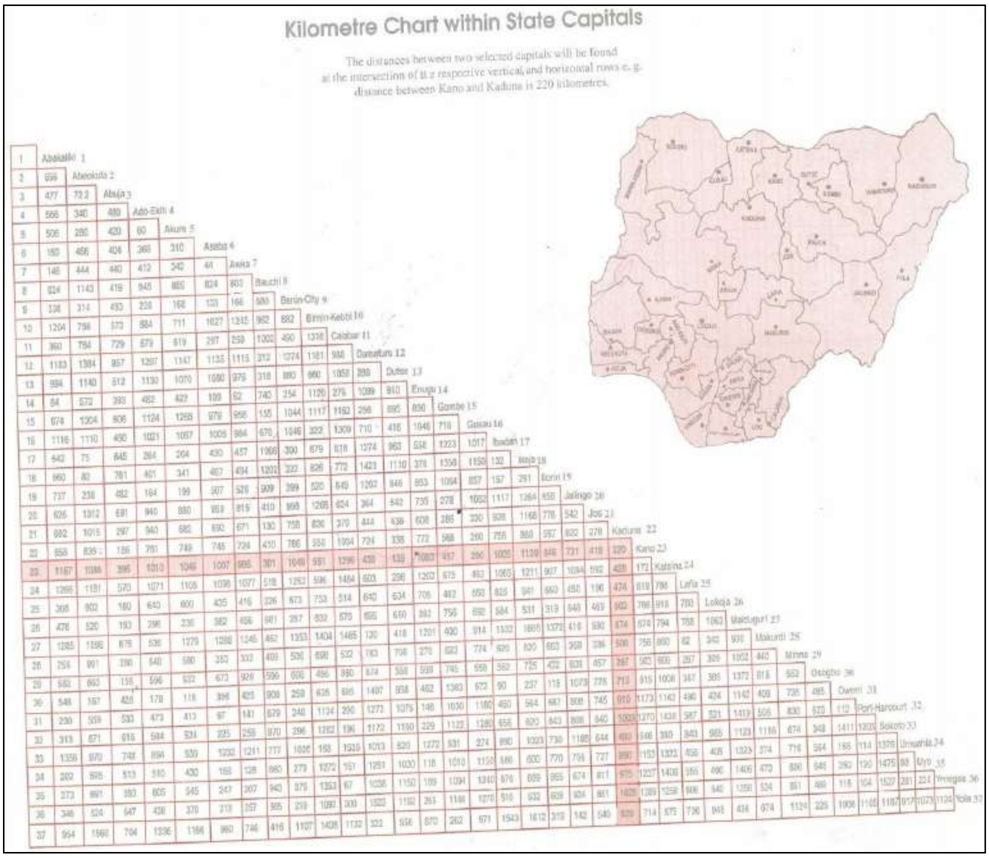
Most traditional path finding solutions are based on shortest path algorithm that tend to minimize the cost of travel from one point to another. Majority of work in network tomography have revolved on active problem from a single source. It has been noted that the problem of identifying a multiple tomography amount to more than just matching nodes with the same label (Rabbat, 2004) [11].

The shortest path problem is a problem of finding the shortest path or route from a starting point to a final destination. Generally, in order to represent the shortest path problem we use graphs. A graph is a mathematical abstract object, which contains sets of vertices and edges. Edges connect pairs of vertices. Along the edges of a graph it is possible to walk by moving from one vertex to other vertices. Depending on whether or not one can walk along the edges by both sides or by only one side determines if the graph is a directed graph or an undirected graph. In addition, lengths of edges are often called weights, and the weights are normally used for calculating the shortest path from one point to another point (Kairanbay and Hajar, 2013) [5]. The shortest path of a network consisting of few nodes and arcs can be obtained with simple arithmetic. However, larger or complex networks can hardly be obtained with simple arithmetic, hence, the need for algorithm (Oladejo and Tamber, 2014) [9]. According to Wang *et al*. (2014) [12], there are quite a number of existing shortest path search algorithms designed for shortest path problem. These include: Dynamic programming, A* search, Dijkstra, Bellman-Ford-Moore, Floyd-Warshall, Johnson, Veterbi and linear programming

Ahuja *et al.* (1995) [1], presented 42 applications of shortest path problem drawn from the fields of operations research, computer science, physical sciences, medicine, engineering and applied mathematics, some of which are: Telephone operator scheduling, Matrix rounding problem, Locating object in space, Rewiring of typewriter, Dual completion of oil wells, Urban traffic flows, Tanker scheduling, Cluster analysis, Determining an optimal energy policy, Routing of multiple commodities, Local access telephone network capacity expansion, Multi-item production planning, among others.

Data collection

Information and data captured were appropriately analyzed using the data from the kilometer chart of state capital in Nigeria as shown in Figure 1 and Nigeria road network map as shown in Figure 2. The data were transformed into the network model of multiple sources-multiple destinations as shown in Figure 3. The data were used as positive edge weight based on current road distances between state capitals to develop the linear programming problem.



Source: Fidelity Bank Plc Dairy, 2010

Fig 1: Kilometer Chart of State Capital in Nigeria

Fig 2: Nigeria Roads Network System (source: Wikipedia, 2016)

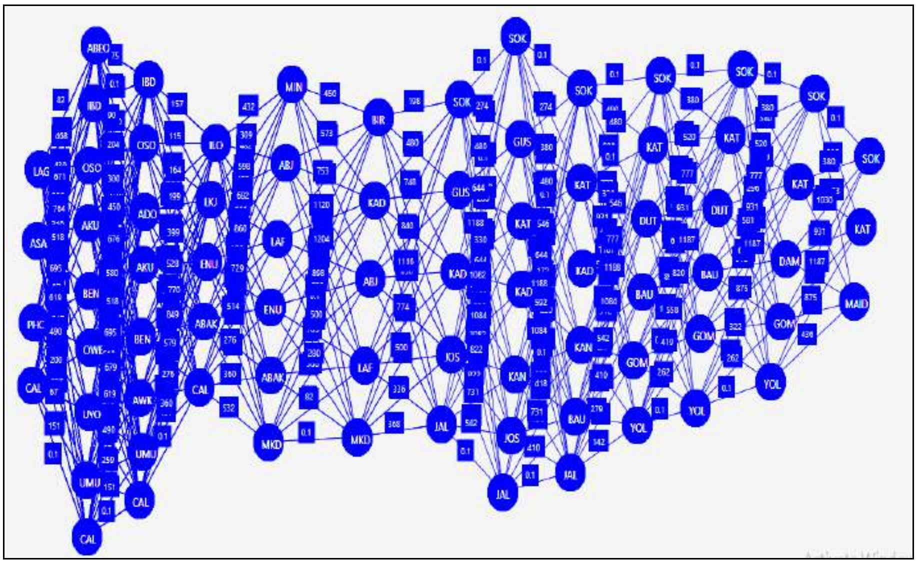


Fig 3: Nigeria road network of motorable road from multiple sources to multiple destinations) with distance measurement (km)

Notation and Abbreviation

From Figure 3: LAG = Lagos; ASA= Asaba; PHC= Port Harcourt; CAL= Calabar; ABEO= Abeokuta; IBD= Ibadan; OSO= Osogbo; AKU= Akure; BEN= Benin; OW= Owerri; UYO= Uyo; UMU= Umauhia; ADO= Ado-ekiti; AWKA= Awka; ILO= Ilorin; LKJ= Lokoja; ENU= Enugu; ABA= Abkiliki; MIN= Minna; ABJ= Abuja; LAF=Lafia; MKD= Makurdi; BIR= Brinin-Kebbi; KAD= Kaduna; SOK= Sokoto; GUS= Gusau; JOS= Jos; JAL= Jalingo; KAT= Katsina; KANO= Kano; BAU= Bauchi; DUT= Dutse; GOM= Gombe; YOL= Yola and MAID= Maiduguri.

Mathematical Formulation of the Problem

According to Ekoko (2006) [4] and Zainb *et al*. (2019) [13], formulating the linear programme for solving the shortest path from *s-t t*, we choose one variable per edge, x_e. If x_e is picked, $x_e = 1$ else $x_e = 0$ and w_e is the value of the weight of edge *e*. According to him, the cost function is:

Minimize $\sum\sum_{(u,v)\ in\ network} W_{uv}X_{uv}$ (1)

Subject to

$$\sum_{(k,v) in\ network} X_{kv} - \sum_{(u,k) in\ network} X_{uk} = \begin{cases} 1,\ if\ k = s(source) & (2) \\ 0\ for\ all\ other\ k & (3) \\ -1, if\ k = t\ (sink) & (4) \end{cases}$$

$0 \leq x_{uv} \leq 1$ and integer $\forall\ (u, v)$ in network (5)

Equation (2) implies that, the difference in the number of edges leaving source node *s* and entering into source node *s* is 1. Equation (3) implies also, that, for all other nodes, edges leaving them is equal the edges entering into them. Equation (4) means that the difference in the number of edges entering into destination node *t* and leaving destination node *t* is 1. Equation (5) is the non-negativity and integrality requirement.

The LP model of Nigeria road network of multiple sources to multiple destinations was developed by subjecting Figure 3 data into the general model by Ekoko (2006) [4] and Zainb *et al*. (2019) [13] as:

Minimize $82X_{1,5} + 132X_{1,6} + 237X_{1,7} + 341X_{1,8} + 322X_{1,9} + 564X_{1,10} + 689X_{1,11} + 600X_{1,12} + 772X_{1,13} + 468X_{2,5} + 430X_{2,6} + 396X_{2,7} + 310_{2,8} + 133X_{2,9} + 97X_{2,10} + 247X_{2,11} + 166X_{2,12} + 297X_{2,13} + 671X_{3,5} + 656X_{3,6} + 623X_{3,7} + 524X_{3,8} + 296X_{3,9} + 112X_{3,10} + 120X_{3,11} + 114X_{3,12} + 196X_{3,13} + 764X_{4,5} + 518X_{4,6} + 695X_{4,7} + 619X_{4,8} + 490X_{4,9} + 200X_{4,10} + 67X_{4,11} + 151X_{4,12} + 0X_{4,13} + 75X_{5,14} + 167X_{5,15} + 340X_{5,16} + 280X_{5,17} + 314X_{5,18} + 444_{5,19} + 595X_{5,20} + 764X_{5,21} + 0X_{6,14} + 90X_{6,15} + 264X_{6,16} + 204X_{6,17} + 300X_{6,18} + 457X_{6,19} + 580X_{6,20} + 518X_{6,21} + 90X_{7,14} + 0X_{7,15} + 178X_{7,16} + 118X_{7,17} + 259X_{7,18} + 425X_{7,19}\ 564X_{7,20} + 695X_{7,21} + 204X_{8,14} + 118X_{8,15} + 60X_{8,16} + 0X_{8,17} + 711X_{8,18} + 342X_{8,19} + 430X_{8,20} + 619X_{8,21} + 300X_{9,14} + 259X_{9,15} + 228X_{9,16} + 168X_{9,17} + 0X_{9,18} + 166X_{9,19} + 279X_{9,20} + 490X_{9,21} + 450X_{10,14} + 495X_{10,15} + 473X_{10,16} + 413X_{10,17} + 240X_{10,18} + 141X_{10,19} + 165X_{10,20} + 200X_{10,21} + 676X_{11,14} + 645X_{11,15} + 606X_{11,16} + 545X_{11,17} + 375X_{11,18} + 207X_{11,19} + 88X_{11,20} + 67X_{11,21} + 580X_{12,14} + 564X_{12,15} + 510X_{12,16} + 430X_{12,17} + 279X_{12,18} + 126X_{12,19} + 0X_{12,20} + 151X_{12,21} + 518X_{13,14} + 695X_{13,15} + 679X_{13,16} + 619X_{13,17} + 490X_{13,18} + 259X_{13,19} + 151X_{13,20} + 0X_{13,21} + 157X_{14,22} + 584X_{14,23} + 558X_{14,24} + 642X_{14,25} + 518X_{14,26} + 115X_{15,22} + 305X_{15,23} + 462X_{15,24} + 546X_{15,25} + 695X_{15,26} + 164X_{16,22} + 296X_{16,23} + 482X_{16,24} + 566X_{16,25} + 679X_{16,26} + 199X_{17,22} + 236X_{17,23} + 422X_{17,24} + 506X_{17,25} + 619X_{17,26} + 399X_{18,22} + 287X_{18,23} + 257X_{18,24} + 338X_{18,25} + 490X_{18,26} + 528X_{19,22} + 456X_{19,23} + 62X_{19,24} + 146X_{19,25} + 259X_{19,26} + 770X_{20,22} + 405X_{20,23} + 118X_{20,24} + 202X_{20,25} + 151X_{20,26} + 849X_{21,22} + 579X_{21,23} + 276X_{21,24} + 360X_{21,25} + 0X_{21,26} + 432X_{22,27} + 482X_{22,28} + 660X_{22,29} + 653X_{22,30} + 660X_{22,31} + 863X_{22,32} + 309X_{23,27} + 193\ X_{23,28} + 788\ X_{23,29} + 392\ X_{23,30} + 476\ X_{23,31} + 342\ X_{23,32} + 598X_{24,27} + 393\ X_{24,28} + 705\ X_{24,29} + 0\ X_{24,30} + 84\ X_{24,31} + 270\ X_{24,32} + 682X_{25,27} + 477X_{25,28} + 388X_{25,29} + 84X_{25,30} + 0X_{25,31} + 256X_{25,32} + 860X_{26,27} + 729\ X_{26,28} + 514\ X_{26,29} + 276\ X_{26,30} + 360\ X_{26,31} + 532\ X_{26,32} + 456X_{27,33} + 297\ X_{27,34} + 156\ X_{27,35} + 682X_{27,36} + 440X_{27,37} + 573X_{28,33} + 186X_{28,34} + 0X_{28,35} + 180X_{28,36} + 280X_{28,37} + 753X_{29,33} + 474X_{29,34} + 180X_{29,35} + 0X_{29,36} + 82X_{29,37} + 1120X_{30,33} + 772X_{30,34} + 393X_{30,35} + 705X_{30,36} + 270X_{30,37} + 1204X_{31,33} + 856X_{31,34} + 477X_{31,35} + 388X_{31,36} + 256X_{31,37} + 898X_{32,33} + 500X_{32,34} + 280X_{32,35} + 82X_{32,36} + 0X_{32,37} + 198X_{33,38} + 322X_{33,39} + 550X_{33,40} + 830X_{33,41} + 1266X_{33,42} + 480X_{34,38} + 260X_{34,39} + 0X_{34,40} + 278X_{34,41} + 822X_{34,42} + 748X_{35,38} + 490X_{35,39} + 186X_{35,40} + 297X_{35,41} + 691X_{35,42} + 840X_{36,38} + 650X_{36,39} + 474X_{36,40} + 196X_{36,41} + 450X_{36,42} + 1116X_{37,38} + 774X_{37,39} + 500X_{37,40} + 336X_{37,41} + 368X_{37,42} + 0X_{38,43} + 274X_{38,44} + 380X_{38,45} + 480X_{38,46} + 546X_{38,47} + 644X_{38,48} + 1188X_{38,49} + 274X_{39,43} + 0X_{39,44} + 463X_{39,45} + 260X_{39,46} + 260X_{39,47} + 330X_{39,48} + 1082X_{39,49} + 480X_{40,43} + 260X_{40,44} + 428X_{40,45} + 0X_{40,46} + 220X_{40,47} + 278X_{40,48} + 822X_{40,49} + 644X_{41,43} + 330X_{41,44} + 592X_{41,45} + 278X_{41,46} + 418X_{41,47} + 0X_{41,48} + 542X_{41,49} + 1188X_{42,43} + 1082X_{42,44} + 1084X_{42,45} + 822X_{42,46} + 731X_{42,47} + 542X_{42,48} + 0X_{42,49} + 0X_{43,50} + 380X_{43,51} + 546X_{43,52} + 777X_{43,53} + 1188X_{43,54} + 1188X_{43,55} + 274X_{44,50} + 463X_{44,51} + 260X_{44,52} + 670X_{44,53} + 1082X_{44,54} + 1082X_{44,55} + 380X_{45,50} + 0X_{45,51} + 172X_{45,52} + 518X_{45,53} + 1084X_{45,54} + 1084X_{45,55} + 480X_{46,50} + 428X_{46,51} + 220X_{46,52} + 410X_{46,53} + 822X_{46,54} + 822X_{46,55} + 546X_{47,50} + 172X_{47,51} + 0X_{47,52} + 301X_{47,53} + 731X_{47,54} + 731X_{47,55} + 644X_{48,50} + 592X_{48,51} + 418X_{48,52} + 130X_{48,53} + 54X_{48,54} + 54X_{48,55} + 1188X_{49,50} + 1084X_{49,51} + 731X_{49,52} + 410X_{49,53} + 0X_{49,54} + 0X_{49,55} + 0X_{50,56} + 480X_{50,57} + 520X_{50,58} + 777X_{50,59} + 931X_{50,60} + 1187X_{50,61} + 0X_{51,56} + 480X_{51,57} + 520X_{51,58} + 777X_{51,59} + 931X_{51,60} + 1187X_{51,61} + 480X_{52,56} + 0X_{52,57} + 338X_{52,58} + 410X_{52,59} + 566X_{52,60} + 875X_{52,61} + 546X_{53,56} + 172X_{53,57} + 136X_{53,58} + 301X_{53,59} + 457X_{53,60} + 714X_{53,61} + 777X_{54,56} + 516X_{54,57} + 312X_{54,58} + 0X_{54,59} + 155X_{54,60} + 416X_{54,61} + 1188X_{55,56} + 1084X_{55,57} + 542X_{55,58} + 410X_{55,59} + 279X_{55,60} + 142X_{55,61} + 0X_{56,62} + 380X_{56,63} + 520X_{56,64} + 777X_{56,65} + 931X_{56,66} + 1187X_{56,67} + 380X_{57,62} + 0\ X_{57,63} + 296X_{57,64} + 518X_{57,65} + 675X_{57,66} + 820X_{57,67} + 520X_{58,62} + 296X_{58,63} + 0X_{58,64} + 318X_{58,65} + 895X_{58,66} + 558X_{58,67} + 777X_{59,62} + 518X_{59,63} + 318X_{59,64} + 0X_{59,65} + 155X_{59,66} + 416X_{59,67} + 931X_{60,62} + 675X_{60,63} + 895X_{60,64} + 155X_{60,65} + 0\ X_{60,66} + 262X_{60,67} + 1187X_{61,62} + 820X_{61,63} + 558X_{61,64} + 416X_{61,65} + 262X_{61,66} + 0X_{61,67} + 0X_{62,68} + 380X_{62,69} + 1030X_{62,70} + 931X_{62,71} + 1187X_{62,72} + 380X_{63,68} + 0X_{63,69} + 603X_{63,70} + 671X_{63,71} + 875X_{63,72} + 520X_{64,68} + 296X_{64,69} + 288X_{64,70} + 895X_{64,71} + 558X_{64,72} + 777X_{65,68} + 581X_{65,69} + 312X_{65,70} + 155X_{65,71} + 416X_{65,72} + 931X_{66,68} + 675X_{66,69} + 256X_{66,70} + 0X_{66,71} + 262X_{66,72} + 1187X_{67,68} + 875X_{67,69} + 322X_{67,70} + 262X_{67,71} + 0X_{67,72} + 0X_{68,73} + 380X_{68,74} + 1123X_{68,75} + 0X_{68,76} + 380X_{69,73} + 0X_{69,74} + 794X_{69,75} + 0X_{69,76} + 1030X_{70,73} + 603X_{70,74} + 130X_{70,75} + 0X_{70,76} + 931X_{71,73} + 675X_{71,74} + 400X_{71,75} + 0X_{71,76} + 1187X_{72,73} + 875X_{72,74} + 436X_{72,75} + 0X_{72,76}$

Subject to

$X_{1,5} + X_{1,6} + X_{1,7} + X_{1,8} + X_{1,9} + X_{1,10} + X_{1,11} + X_{1,12} + X_{1,13} \leq 1;$

$X_{2,5} + X_{2,6} + X_{2,7} + X_{2,8} + X_{2,9} + X_{2,10} + X_{2,11} + X_{2,12} + X_{2,13} \leq 1;$

$X_{3,5} + X_{3,6} + X_{3,7} + X_{3,8} + X_{3,9} + X_{3,10} + X_{3,11} + X_{3,12} + X_{3,13} \leq 1;$

$X_{4,5} + X_{4,6} + X_{4,7} + X_{4,8} + X_{4,9} + X_{4,10} + X_{4,11} + X_{4,12} + X_{4,13} \leq 1;$

$X_{68,73} + X_{69,73} + X_{70,73} + X_{71,73} + X_{72,73} \geq 1;$

$X_{68,74} + X_{69,74} + X_{70,74} + X_{71,74} + X_{72,74} \geq 1;$

$X_{68,75} + X_{69,75} + X_{70,75} + X_{71,75} + X_{72,75} \geq 1;$

$X_{68,76} + X_{69,76} + X_{70,76} + X_{71,76} + X_{72,76} \geq 1;$

$X_{1,5} + X_{2,5} + X_{3,5} + X_{4,5} = X_{5,14} + X_{5,15} + X_{5,16} + X_{5,17} + X_{5,18} + X_{5,19} + X_{5,20} + X_{5,21} + X_{6,14};$

$X_{1,6} + X_{2,6} + X_{3,6} + X_{4,6} = X_{6,15} + X_{6,16} + X_{6,17} + X_{6,18} + X_{6,19} + X_{6,20} + X_{6,21};$

$X_{1,7} + X_{2,7} + X_{3,7} + X_{4,7} = X_{7,14} + X_{7,15} + X_{7,16} + X_{7,17} + X_{7,18} + X_{7,19} + X_{7,20} + X_{7,21};$

$X_{1,8} + X_{2,8} + X_{3,8} + X_{4,8} = X_{8,14} + X_{8,15} + X_{8,16} + X_{8,17} + X_{8,18} + X_{8,19} + X_{8,20} + X_{8,21};$

$X_{1,9} + X_{2,9} + X_{3,9} + X_{4,9} = X_{9,14} + X_{9,15} + X_{9,16} + X_{9,17} + X_{9,18} + X_{9,19} + X_{9,20} + X_{9,21};$

$X_{1,10} + X_{2,10} + X_{3,10} + X_{4,10} = X_{10,14} + X_{10,15} + X_{10,16} + X_{10,17} + X_{10,18} + X_{10,19} + X_{10,20} + X_{10,21};$

$X_{1,11} + X_{2,11} + X_{3,11} + X_{4,11} = X_{11,14} + X_{11,15} + X_{11,16} + X_{11,17} + X_{11,18} + X_{11,19} + X_{11,20} + X_{11,21};$

$X_{1,12} + X_{2,12} + X_{3,12} + X_{4,12} = X_{12,14} + X_{12,15} + X_{12,16} + X_{12,17} + X_{12,18} + X_{12,19} + X_{12,20} + X_{12,21};$ $X_{1,13} + X_{2,13} + X_{3,13} + X_{4,13} = X_{13,14} + X_{13,15} + X_{13,16} + X_{13,17} + X_{13,18} + X_{13,19} + X_{13,20} + X_{13,21};$

$X_{5,14} + X_{6,14} + X_{7,14} + X_{8,14} + X_{9,14} + X_{10,14} + X_{11,14} + X_{12,14} + X_{13,14} = X_{14,22} + X_{14,23} + X_{14,24} + X_{14,25} + X_{14,26};$

$X_{5,15} + X_{6,15} + X_{7,15} + X_{8,15} + X_{9,15} + X_{10,15} + X_{11,15} + X_{12,15} + X_{13,15} = X_{15,22} + X_{15,23} + X_{15,24} + X_{15,25} + X_{15,26};$

$X_{5,16} + X_{6,16} + X_{7,16} + X_{8,16} + X_{9,16} + X_{10,16} + X_{11,16} + X_{12,16} + X_{13,16} = X_{16,22} + X_{16,23} + X_{16,24} + X_{16,25} + X_{16,26};$

$X_{5,17} + X_{6,17} + X_{7,17} + X_{8,17} + X_{9,17} + X_{10,17} + X_{11,17} + X_{12,17} + X_{13,17} = X_{17,22} + X_{17,23} + X_{17,24} + X_{17,25} + X_{17,26};$

$X_{5,19} + X_{6,19} + X_{7,19} + X_{8,19} + X_{9,19} + X_{10,19} + X_{11,19} + X_{12,19} + X_{13,19} = X_{19,22} + X_{19,23} + X_{19,24} + X_{19,25} + X_{19,26};$

$X_{5,20} + X_{6,20} + X_{7,20} + X_{8,20} + X_{9,20} + X_{10,20} + X_{11,20} + X_{12,20} + X_{13,20} = X_{20,22} + X_{20,23} + X_{20,24} + X_{20,25} + X_{20,26};$

$X_{5,21} + X_{6,21} X_{7,21} + X_{8,21} + X_{9,21} + X_{10,21} + X_{11,21} + X_{12,21} + X_{13,21} = X_{21,22} + X_{21,23} + X_{21,24} + X_{21,25} + X_{21,26};$

$X_{14,22} + X_{15,22} + X_{16,22} + X_{17,22} + X_{18,22} + X_{19,22} + X_{20,22} + X_{21,22} = X_{22,27} + X_{22,28} + X_{22,29} + X_{22,30} + X_{22,31} + X_{22,32};$

$X_{14,23} + X_{15,23} + X_{16,23} + X_{17,23} + X_{18,23} + X_{19,23} + X_{20,23} + X_{21,23} = X_{23,27} + X_{23,28} + X_{23,29} + X_{23,30} + X_{23,31} + X_{23,32};$

$X_{14,24} + X_{15,24} + X_{16,24} + X_{17,24} + X_{18,24} + X_{19,24} + X_{20,24} + X_{21,24} = X_{24,27} + X_{24,28} + X_{24,29} + X_{24,30} + X_{24,31} + X_{24,32};$

$X_{14,25} + X_{15,25} + X_{16,25} + X_{17,25} + X_{18,25} + X_{19,25} + X_{20,25} + X_{21,25} = X_{25,27} + X_{25,28} + X_{25,29} + X_{25,30} + X_{25,31} + X_{25,32};$

$X_{14,26} + X_{15,26} + X_{16,26} X_{17,26} + X_{18,26} + X_{19,26} + X_{20,26} + X_{21,26} = X_{26,27} + X_{26,28} + X_{26,29} + X_{26,30} + X_{26,31} + X_{26,32};$

$X_{22,27} + X_{23,27} + X_{24,27} + X_{25,27} + X_{26,27} = X_{27,33} + X_{27,34} + X_{27,35} + X_{27,36} + X_{27,37};$

$X_{22,28} + X_{23,28} + X_{24,28} + X_{25,28} + X_{26,28} = X_{28,33} + X_{28,34} + X_{28,35} + X_{28,36} + X_{28,37};$

$X_{22,29} + X_{23,29} + X_{24,29} + X_{25,29} + X_{26,29} = X_{29,33} + X_{29,34} + X_{29,35} + X_{29,36} + X_{29,37};$

$X_{22,30} + X_{23,30} + X_{24,30} + X_{25,30} + X_{26,30} = X_{30,33} + X_{30,34} + X_{30,35} + X_{30,36} + X_{30,37};$

$X_{22,31} + X_{23,31} + X_{24,31} + X_{25,31} + X_{26,31} = X_{31,33} + X_{31,34} + X_{31,35} + X_{31,36} + X_{31,37};$

$X_{22,32} + X_{23,32} + X_{24,32} + X_{25,32} + X_{26,32} = X_{32,33} + X_{32,34} + X_{32,35} + X_{32,36} + X_{32,37};$

$X_{27,33} + X_{28,33} + X_{29,33} + X_{30,33} + X_{31,33} + X_{32,33} = X_{33,38} + X_{33,39} + X_{33,40} + X_{33,41} + X_{33,42};$

$X_{27,34} + X_{28,34} + X_{29,34} + X_{30,34} + X_{31,34} + X_{32,34} = X_{34,38} + X_{34,39} + X_{34,40} + X_{34,41} + X_{34,42};$

$X_{27,35} + X_{28,35} + X_{29,35} + X_{30,35} + X_{31,35} + X_{32,35} = X_{35,38} + X_{35,39} + X_{35,40} + X_{35,41} + X_{35,42};$

$X_{27,36} + X_{28,36} + X_{29,36} + X_{30,36} + X_{31,36} + X_{32,36} = X_{36,38} + X_{36,39} + X_{36,40} + X_{36,41} + X_{36,42};$

$X_{27,37} + X_{28,37} + X_{29,37} + X_{30,37} + X_{31,37} + X_{32,37} = X_{37,38} + X_{37,39} + X_{37,40} + X_{37,41} + X_{37,42};$

$X_{33,38} + X_{34,38} + X_{35,38} + X_{36,38} + X_{37,38} = X_{38,43} + X_{38,44} + X_{38,45} + X_{38,46} + X_{38,47} + X_{38,48} + X_{38,49};$

$X_{33,39} + X_{34,39} + X_{35,39} + X_{36,39} + X_{37,39} = X_{39,43} + X_{39,44} + X_{39,45} + X_{39,46} + X_{39,47} + X_{39,48} + X_{39,49};$

$X_{33,40} + X_{34,40} + X_{35,40} + X_{36,40} + X_{37,40} = X_{40,43} + X_{40,44} + X_{40,45} + X_{40,46} + X_{40,47} + X_{40,48} + X_{40,49};$

$X_{33,41} + X_{34,41} + X_{35,41} + X_{36,41} + X_{37,41} = X_{41,43} + X_{41,44} + X_{41,45} + X_{41,46} + X_{41,47} + X_{41,48} + X_{41,49};$

$X_{33,42} + X_{34,42} + X_{35,42} + X_{36,42} + X_{37,42} = X_{42,43} + X_{42,44} + X_{42,45} + X_{42,46} + X_{42,47} + X_{42,48} + X_{42,49};$

$X_{38,43} + X_{39,43} + X_{40,43} + X_{41,43} + X_{42,43} = X_{43,50} + X_{43,51} + X_{43,52} + X_{43,53} + X_{43,54} + X_{43,55};$

$X_{38,44} + X_{39,44} + X_{40,44} + X_{41,44} + X_{42,44} = X_{44,50} + X_{44,51} + X_{44,52} + X_{44,53} + X_{44,54} + X_{44,55};$

$X_{38,45} + X_{39,45} + X_{40,45} + X_{41,45} + X_{42,45} = X_{45,50} + X_{45,51} + X_{45,52} + X_{45,53} + X_{45,54} + X_{45,55};$

$X_{38,46} + X_{39,46} + X_{40,46} + X_{41,46} + X_{42,46} = X_{46,50} + X_{46,51} + X_{46,52} + X_{46,53} + X_{46,54} + X_{46,55};$

$X_{38,47} + X_{39,47} + X_{40,47} + X_{41,47} + X_{42,47} = X_{47,50} + X_{47,51} + X_{47,52} + X_{47,53} + X_{47,54} + X_{47,55};$

$X_{38,48} + X_{39,48} + X_{40,48} + X_{41,48} + X_{42,48} = X_{48,50} + X_{48,51} + X_{48,52} + X_{48,53} + X_{48,54} + X_{48,55};$

$X_{38,49} + X_{39,49} + X_{40,49} + X_{41,49} + X_{42,4} = X_{49,50} + X_{49,51} + X_{49,52} + X_{49,53} + X_{49,54} + X_{49,55};$

$X_{43,50} + X_{44,50} + X_{45,50} + X_{46,50} + X_{47,50} + X_{48,50} + X_{49,50} = X_{50,56} + X_{50,57} + X_{50,58} + X_{50,59} + X_{50,60} + X_{50,61};$

$X_{43,51} + X_{44,51} + X_{45,51} + X_{46,51} + X_{47,51} + X_{48,51} + X_{49,51} = X_{51,56} + X_{51,57} + X_{51,58} + X_{51,59} + X_{51,60} + X_{51,61};$

$X_{43,52} + X_{44,52} + X_{45,52} + X_{46,52} + X_{47,52} + X_{48,52} + X_{49,52} = X_{52,56} + X_{52,57} + X_{52,58} + X_{52,59} + X_{52,60} + X_{52,61};$

$X_{43,53} + X_{44,53} + X_{45,53} + X_{46,53} + X_{47,53} + X_{48,53} + X_{49,53} = X_{53,56} + X_{53,57} + X_{53,58} + X_{53,59} + X_{53,60} + 714X_{53,61};$

$X_{43,54} + X_{44,54} + X_{45,54} + X_{46,54} + X_{47,54} + X_{48,54} + X_{49,54} = X_{54,56} + X_{54,57} + X_{54,58} + X_{54,59} + X_{54,60} + X_{54,61};$

$X_{43,55} + X_{44,55} + X_{45,55} + X_{46,55} + X_{47,55} + X_{48,55} + X_{49,55} = X_{55,56} + X_{55,57} + X_{55,58} + X_{55,59} + X_{55,60} + X_{55,61};$

$X_{51,56} + X_{52,56} + X_{53,56} + X_{54,56} + X_{55,56} = X_{56,62} + X_{56,63} + X_{56,64} + X_{56,65} + X_{56,66} + X_{56,67};$

$X_{51,57} + X_{52,57} + X_{53,57} + X_{54,57} + X_{55,57} = X_{57,62} + X_{57,63} + X_{57,64} + X_{57,65} + X_{57,66} + X_{57,67};$

$X_{51,58} + X_{52,58} + X_{53,58} + X_{54,58} + X_{55,58} = X_{58,62} + X_{58,63} + X_{58,64} + X_{58,65} + X_{58,66} + X_{58,67};$

$X_{51,59} + X_{52,59} + X_{53,59} + X_{54,59} + X_{55,59} = X_{59,62} + X_{59,63} + X_{59,64} + X_{59,65} + X_{59,66} + X_{59,67};$

$X_{51,60} + X_{52,60} + X_{53,60} + X_{54,60} + X_{55,60} = X_{60,62} + X_{60,63} + X_{60,64} + X_{60,65} + X_{60,66} + X_{60,67};$

$X_{51,61} + X_{52,61} + X_{53,61} + X_{54,61} + X_{55,61} = X_{61,62} + X_{61,63} + X_{61,64} + X_{61,65} + X_{61,66} + X_{61,67};$

$X_{56,62} + X_{57,62} + X_{58,62} + X_{59,62} + X_{60,62} + X_{61,62} = X_{62,68} + X_{62,69} + X_{62,70} + X_{62,71} + X_{62,72};$

$X_{56,63} + X_{57,63} + X_{58,63} + X_{59,63} + X_{60,63} + X_{61,63} = X_{63,68} + X_{63,69} + X_{63,70} + X_{63,71} + X_{63,72};$

$X_{56,64} + X_{57,64} + X_{58,64} + X_{59,64} + X_{60,64} + X_{61,64} = X_{64,68} + X_{64,69} + X_{64,70} + X_{64,71} + X_{64,72};$

$X_{56,65} + X_{57,65} + X_{58,65} + X_{59,65} + X_{60,65} + X_{61,65} = X_{65,68} + X_{65,69} + X_{65,70} + X_{65,71} + X_{65,72};$

$X_{56,66} + X_{57,66} + X_{58,66} + X_{59,66} + X_{60,66} + X_{61,66} = X_{66,68} + X_{66,69} + X_{66,70} + X_{66,71} + X_{66,72};$

$X_{56,67} + X_{57,67} + X_{58,67} + X_{59,67} + X_{60,67} + X_{61,67} = X_{67,68} + X_{67,69} + X_{67,70} + X_{67,71} + X_{67,72};$

$X_{62,68} + X_{63,68} + X_{64,68} + X_{65,68} + X_{66,68} + X_{67,68} = X_{68,73} + X_{68,74} + X_{68,75} + X_{68,76};$

$X_{62,69} + X_{63,69} + X_{64,69} + X_{65,69} + X_{66,69} + X_{67,69} = X_{69,73} + X_{69,74} + X_{69,75} + X_{69,76};$
$X_{62,70} + X_{63,70} + X_{64,70} + X_{65,70} + X_{66,70} + X_{67,70} = X_{70,73} + X_{70,74} + X_{70,75} + X_{70,76};$
$X_{62,71} + X_{63,71} + X_{64,71} + X_{65,71} X_{66,71} + X_{67,71} = X_{71,73} + X_{71,74} + X_{71,75} + X_{71,76};$
$X_{62,72} + X_{63,72} + X_{64,72} + X_{65,72} + X_{66,72} + X_{67,72} = X_{72,73} + X_{72,74} + X_{72,75} + X_{72,76};$
$0 \le X_{uv} \le 1$ and integers. $u = 1, 2, 3 \ldots 72, v = 1, 2, 3 \ldots 76.$

Result

Subjecting the objective function and 75 constraints with 441 variables above to LINGO computer application package which was developed by LINDO System Inc (LINGO, 2013) [6], the shortest route from the multiple sources (Lagos, Asaba, Port Harcourt and Calabar) to multiple destinations (Sokoto, Katsina and Maiduguri) are as shown in Figure 4 and listed in Table 1.

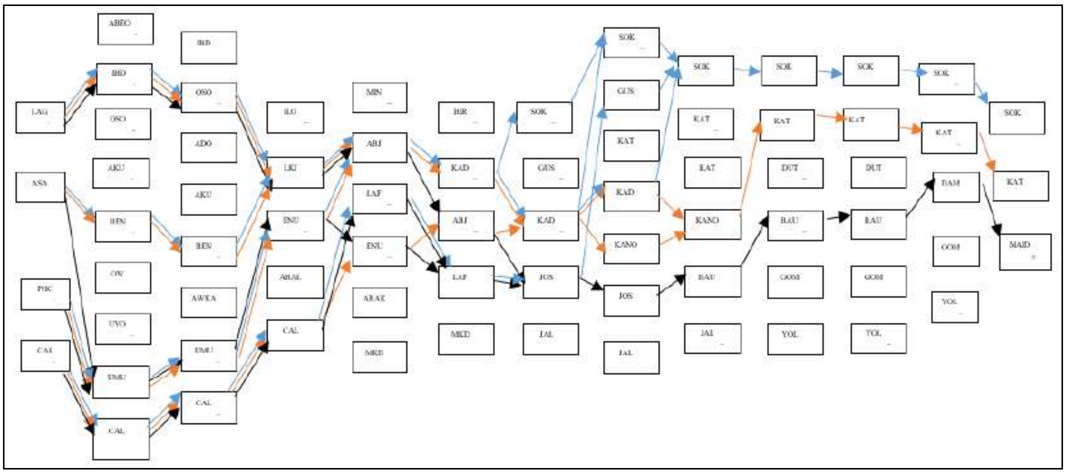


Fig 4: The result of shortest paths from multiple sources to multiple destinations

Discussion

In this research work, we developed a model of Nigeria road network from multiple sources to multiple destinations through several intermediary nodes (states) and formulated it as a linear programming problem which has 441 variables and 75 constraints. Due to the large number of variables and constraints, it will be difficult to solve this manually. Hence we employed the use of LINGO computer application package which solves any LPP directly. The solutions in Figure 4 above are depicted in Table 1.

Table 1: The shortest routes from the coastal town to border towns

Source	Destination	Route	Total Distance (KM)
Lagos	Sokoto	Lagos-Ibadan-Oshogbo-Lokoja-Abuja-Kaduna Sokoto	1375
Lagos	Katsina	Lagos- Ibadan-Oshogbo-Lokoja-Abuja-Kaduna-Kano-Katsina	1298
Lagos	Maiduguri	Lagos- Ibadan-Oshogbo-Lokoja-Abuja-Jos-Bauchi-Damaturu-Maiduguri	1589
Asaba/Warri	Sokoto	Asaba-Benin-Lokoja-Abuja-Kaduna-Sokoto	1279
Asaba/Warri	Katsina	Asaba-Benin-Lokoja-Abuja-Kaduna-Kano-Katsina	1191
Asaba/Warri	Maiduguri	Asaba-Umuahia-Enugu-Lafia-Jos- Bauchi-Damaturu- Maiduguri	1404
Port Harcourt	Sokoto	PHC-Umuahia-Enugu- Abuja-Kaduna-Sokoto	1291
Port Harcourt	Katsina	PHC-Umuahia-Enugu- Abuja-Kaduna-Kano-Katsina	1203
Port Harcourt	Maiduguri	PHC-Umuahia-Enugu- Lafia-Jos- Bauchi-Damaturu-Maiduguri	1352
Calabar	Sokoto	Calabar-Lafia-Jos-Gusau-Sokoto	1314
Calabar	Katsina	Calabar-Enugu- Abuja-Kaduna-Kano-Katsina	1240
Calabar	Maiduguri	Calabar-Lafia-Jos- Bauchi-Damaturu-Maiduguri	1282

Conclusion

In this paper, we have successfully developed the Nigeria road network model consisting of multiple coastal cities of Lagos, Warri/Asaba, Port-Harcourt and Calabar as sources through intermediary stagecoach cities to multiple destination border cities of Sokoto, Katsina and Maiduguri. We also formulated a mathematical model of the Nigeria road network from multiple sources – multiple destinations using the linear programming problem with 1 objective function, 75 constraints and 441 variables. We obtained a shortest path of the developed a model of the Nigeria road network of multiple sources to multiple destinations.

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