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# Stability analysis and development of gear shift patterns for an eight speed automatic transmission vehicle

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#### Abstract

The increasing stringency of exhaust emission regulations, as well as the rising index of carbon dioxide emissions from the road transport, has driven auto manufacturers and designers to develop solutions for even more efficient vehicles, with well-designed vehicle transmission and gear shift patterns among the alternative approaches used. The current article proposes a design method for developing an appropriate gear ratio set and gear shift patterns for an eight-speed automatic transmission. The BMW M4 Eight speed transmission model was developed using the MATLAB/Simulink platform, and the results were validated using experiment data. The effectiveness of the previously chosen gear ratio set and gear shift patterns was then assessed using a eight-speed transmission model based on the validated six speed transmission model. Based on a mathematical simulation approach, this article presents a novel microscopic model to illustrate the traffic phenomenon. The stability of the proposed model is investigated using the perturbation technique in order to analyze it and determine its stability conditions. The mathematical experimental tests were carried out, and the results reveal that the proposed model is valid. Patterns for sport mode, eco mode, and an accomplished by two modes were analyzed and the results.

**Keywords:** Fuel consumption, automatic transmission, car following model, perturbation method, gear shifting pattern, software simulation model, internal combustion engine

#### Introduction

Internal combustion engines (ICEs) are sometimes ubiquitous in human cultures for decades. vet their exhaust fumes have always been hazardous to the environment. Much research has been done in this area to reduce fuel usage and pollution. Electric hybrid vehicles, which can lower exhaust emissions by up to 25%, are one of the finest alternatives thus far. Furthermore, by combining novel and innovative manufacturing quality of connected mechanical components with optimality in developed algorithms, exhaust emissions will be reduced by up to 14.5 percent [1]. Yong et al., on the other hand, looked into the best gear shifting point in automatic gearbox systems. They looked at how vehicle mass affected the optimal gear shifting position <sup>[1]</sup>. A fuzzy gear-shifting strategy for AMT gearboxes has been proposed by Sakagucci et al. By establishing a three-layer knowledge base, Sakaguchi et al. have improved the reliability of control devices. They demonstrated the efficiency of the proposed approach by applying the built fuzzy controller to shift scheduling in automatic transmission automobiles <sup>[2]</sup>. Yin et al. developed an ideal gear changing technique based on vehicle torque, fuel efficiency, and pollution indices. The proposed technique was demonstrated to be applicable, and it significantly improved the performance of automobiles equipped with a stepped automatic transmission system <sup>[3]</sup>. Another research project aims to design optimal gear shifting strategies for conventional cars, as well as discuss the best fuel economy and environmental trade-off. Ngo et al. focused on the drivability of a fuel efficient gear shift utilizing the DP algorithm <sup>[4]</sup>. In the urban phase of the NEDC cycle, the proposed approach saved 3.6 percent and 4.3 percent of fuel over a hot- and cold-start, respectively, when applied to the tested vehicle. He and colleagues <sup>[5]</sup> proposed a gearshift timetable for automated vehicles. They used Lyapunov statements to show that in the presence of restrictions, restricted hybrid dynamical systems have equilibrium stability. Furthermore, their technique ensured that torque requirement is met at the most efficient gear ratios. In addition to the throttle position and engine speed. In the NEDC cycle, Oglieve et al. tried to reduce brake specific fuel consumption and nitrogen oxide emissions.

Corresponding Author: M Shunmathi Researcher, Thiagarajar College of Engineering, Madurai, Tamil Nadu, India They achieved a 7.5% reduction in BSFC and a 6.75% reduction in NOx emissions in their best-case scenarios. Finally, they found that transmission optimization may be used to minimize fuel consumption and emissions in an effective and costeffective manner <sup>[6]</sup>. Dynamic Programming (DP) strategy for a vehicle Power-Shift Automated Manual Transmission is included. Significant potential fuel was discovered. The proposed design could result in cost savings method <sup>[17]</sup>. Ngo has extensively examined several gear shifting tactics for automotive transmissions in conventional and hybrid electric vehicles in his book Gear Shift Strategies for Automotive Transmissions. He's also introduced new gear shifting map design approaches <sup>[8]</sup>. Developing effective power train gear changing techniques is one of them. As a result of the improved intelligent control algorithms, fuel consumption and exhaust pollutants should be significantly decreased. Various studies have looked into the impact of gear shifting strategy, or simply driving behaviour, on fuel consumption. Fofana et al.<sup>[9]</sup> designed an efficient dual clutch gear-shifting map as a multi-objective optimization problem in order to minimise exhaust emissions while also optimizing driving dynamics. Casavola et al. examined and compared two online gear-shifting techniques for AMT gearboxes on the NEDC cycle: i) Efficient Gear Actuator and ii) Genetic and Fuzzy Algorithm for AMT gearboxes <sup>[10]</sup>. Santiciolli and colleagues attempted to show that the accompanying gear changing method has a significant impact on the performance and fuel consumption of a typical Brazilian vehicle. To accomplish so, they looked at two approaches: the first was based on an ideal gear map as a function of vehicle torque and velocity, and the second was based on an optimization loop performed above the Brazilian standard driving conditions <sup>[11]</sup>. Eckert *et al.* proposed an ideal gear-shifting approach that used an Interactive Adaptive Weight Genetic Algorithm to get the best fuel consumption and driving performance. The tests were carried out during the FTP-72 cycle's urban driving phase. The presented scheme's best-case scenario resulted in a 10.61% fuel savings and a 50.12% gain in driving performance  $^{[12]}$ . Vagg *et al.* presented a procedure for quantifying fuel and CO<sub>2</sub> over a NEDC cycle <sup>[13]</sup>. Miao et al. developed a three-parameter gear-shifting schedule that includes the terrain coefficient as the third component. Based on the findings, they concluded that implementing the proposed timetable reduces fuel use by up to 3.5% <sup>[14]</sup>. In the urban phase of the NEDC cycle, the proposed approach saved 3.6% and 4.3% of fuel over a hot and cold-start, respectively, when applied to the tested vehicle. They enlisted the help of a genetic algorithm (GA) <sup>[15]</sup> in order to increase vehicle driving performance while also lowering fuel consumption. This study proposes an eight speed Lepelletier automated transmission gear ratio configuration and gear shift pattern. According to simulation data, a vehicle with more gear ratios has better fuel economy than one with fewer. The results of the simulation also reveal that the gear shifting pattern has an impact on the vehicle's acceleration, fuel economy, and shifting sensation.

#### Methodology

The BMWM4 2022 8AT provided the basis for the simulation model. For validation, the simulation results were compared to measured data from a chassis dynamometer test. Figure 1 shows a simplified block diagram of a 4 AT vehicle power train model created with the MATLAB/Simulink platform's model-based design method. Figure 1 depicts a detailed schematic diagram of the model. The engine, torque converter, transmission, vehicle body, gear shifting actuator, driver, and fuel economy models are all part of the 8 AT model. The 8AT's base model was built using the validated 8 AT vehicle power train model. An upshift was made in this shift strategy when the BSFC value on a higher gear was less than the current gear. Marking the intersection point of the BSFC curve between the gears determined the shifting points. The upshift speeds corresponding to specific throttle pedal opening signals of 10%, 20% and 30% were obtained from the engine's BSFC contour map plot. By connecting all of the points, an economy upshift line for each gear could be obtained. The economic downshift lines were calculated using the same linear convergence algorithm method as before. It combined the gear shift patterns from sport and eco-mode for this shift strategy. The combination method was usually created to mimic the typical driving behavior and style of a real driver. When low speed driving is required, this combined mode shift pattern can provide the best fuel economy without sacrificing dynamic performance. The shift map was divided in half by the combined mode shift pattern. Eco-mode was used when the throttle pedal opening signal was less than 50%, while sport-mode was used when the throttle pedal opening signal was greater than 50%. The linearity connecting method was used to connect the throttle pedal opening signal between 50% and 100%. The 8 AT vehicle power train model shown in fig. 1 was used to construct the 8 AT vehicle power train model by replacing the transmission model and gear shifting actuator models to 8 AT based.



Fig 1: Auto transmission vehicle model

## Derivation of BMWM4 2022 model (proposed control algorithm)

Most vehicles maintain equal vehicle spacing and travel velocity in the vehicle model. The expression is given below.

$$A_{K} = v_{K}(t) = K_{P}(V(h_{K}(t-t_{2})) - v_{K}(t-t_{1})) + K_{I} \int_{0}^{t} (V(h_{K}(\tau-t_{2})) - v_{K}(\tau-t_{1})) d\tau$$
(1)

$$V(h) = \begin{cases} 0 & h < h_{PD} \\ \frac{v_{max}}{2} (1 - \cos(\pi \frac{h - h_{PD}}{h_{ES} - h_{PD}})) & h_{PD} \le h \le h_{ES} \\ v_{max} & h > h_{ES} \end{cases}$$
(2)

$$A_{K} = v_{K}(t) = K_{P}(V(h_{K}(t-t_{2})) - v_{K}(t-t_{1})) + \kappa(v_{K+1}(t-t_{1}) - v_{K}(t-t_{1})) + \omega(h_{K}(t-t_{2}) - h_{D})$$
(3)

$$X_{K}^{0}(t) = X_{B}K + V(h)t$$
<sup>(4)</sup>

$$X_{K}(t) = Y_{K}(t) + X_{K}^{0}t$$
(5)

$$Y_{K}(t) = X_{K}(t) - X_{K}^{0}t$$
(6)

$$h_{K}(t) = \Delta X_{K}(t) = X_{K+1}(t) - X_{K}(t)$$
(7)

$$=(Y_{K+1}(t)+X_{K+1}^{0}(t))-(Y_{K}(t)+X_{K}^{0}(t))$$
(8)

$$= (Y_{K+1}(t) + h(K+1) + V(h)t) - (Y_{K}(t) + hK + V(h)t)$$
(9)

$$=\Delta Y_{\rm K}(t) + h_{\rm D} \tag{10}$$

$$V'(h) = \frac{V(h + \Delta Y_{K}) - V(h)}{\Delta Y_{K}}$$
(11)

$$\Delta X_{\rm K}(t) = \Delta Y_{\rm K}(t) + X_{\rm B}(t) \tag{12}$$

$$v_{\rm K} = \dot{\mathbf{Y}}_{\rm K} + \mathbf{V}(\mathbf{h}) \tag{13}$$

$$A_{K} = Y_{K}^{\bullet}(t)$$
(14)

$$V(\Delta X_{K}(t)) = V'(h(t))\Delta Y_{K}(t) + V(h(t))$$
(15)

$$\mathbf{Y}_{K}(t) = \mathbf{K}_{P}[\mathbf{V}(\mathbf{h}_{K}(t-t_{2}))\Delta\mathbf{Y}_{K} - \mathbf{Y}_{K}(t-t_{1})] + \kappa \Delta \mathbf{Y}_{K}(t-t_{1}) + \omega \Delta \mathbf{Y}_{K}(t-t_{2})$$
(16)

$$Y_{\rm K}(t-\tau)$$
 as the Fourier form (17)

$$Y_{K}(t-\tau) = Ae^{j\alpha(K+1)+Z(t-\tau)}$$
(18)

Therefore

 $\Delta Y_{K}(t-\tau) = A e^{j\alpha(K+1)+Z(t-\tau)} - A e^{j\alpha i+Z(t-\tau)}$ (19)

$$\Delta Y_{\rm K}(t-\tau) = A(e^{j\alpha} - 1)e^{(j\alpha i + Z(t-\tau))}$$
(20)

Here, the  $\tau = t_1, t_2$ 

$$AZ^{2}e^{(j\alpha i+2i)} = K_{p}[V(h)A(e^{j\alpha}-1)e^{(j\alpha i+Z(t-t_{2}))} - AZe^{(j\alpha i+Z(t-t_{1}))}] + \kappa AZ(e^{j\alpha}-1)e^{(j\alpha i+Z(t-t_{1}))} + \omega A(e^{j\alpha}-1)e^{(j\alpha i+Z(t-t_{2}))}$$
(21)

After simplification we get:

$$Z^{2} = K_{P}[V'(h)(e^{j\alpha} - 1)e^{-Zt_{2}} - Ze^{-Zt_{1}}] + \kappa Z(e^{j\alpha} - 1)e^{-Zt_{2}} + \omega(e^{j\alpha} - 1)e^{-Zt_{2}}$$
(22)

$$e^{-Zt_1} = 1 - Zt_1, e^{-Zt_2} = 1 - Zt_2$$
(23)

Taylor formula:

$$e^{-Zt_1} = 1 - Zt_1, e^{-Zt_2} = 1 - Zt_2$$
(24)

$$Z^{2} = K_{P}[V(h)(e^{j\alpha} - 1)(1 - Zt_{2}) - Z(1 - Zt_{1})]\kappa Z(e^{j\alpha} - 1)(1 - Zt_{1}) + \omega(e^{j\alpha} - 1)(1 - Zt_{2})$$
(25)

$$Z = Z_1(j\alpha) + Z_2(j\alpha)^2 + ...$$
(26)

 $Z^{2} = K_{p}[V'(h)(e^{j\alpha} - l)(l - Z_{1}(j\alpha) + Z_{2}(j\alpha)^{2}t_{2}) - (Z_{1}(j\alpha) + Z_{2}(j\alpha)^{2})(l - Z_{1}(j\alpha) + Z_{2}(j\alpha)^{2}t_{1})] + \kappa(Z_{1}(j\alpha) + Z_{2}(j\alpha)^{2})(e^{j\alpha} - l)(l - Z_{1}(j\alpha) + Z_{2}(j\alpha)^{2}t_{1})] + \alpha((e^{j\alpha} - l)(l - Z_{1}(j\alpha) + Z_{2}(j\alpha)^{2}t$ 

$$e^{t} = 1 + t + \frac{t^{2}}{2!} + \frac{t^{3}}{3!} + \dots$$
(28)

$$[Z_{1}t + Z_{2}t^{2}]^{2} = K_{P}[V(h)(t + \frac{t^{2}}{2})(1 - (Z_{1}t + Z_{2}t^{2})t_{1}) - (Z_{1}t + Z_{2}t^{2})((1 - (Z_{1}t + Z_{2}t^{2})t_{1})] + \kappa(Z_{1}t + Z_{2}t^{2})((1 - (Z_{1}t + Z_{2}t^{2})t_{1}))] + \kappa(Z_{1}t + Z_{2}t^{2})((1 - (Z_{1}t + Z_{2}t^{2})t_{1})))] + \kappa(Z_{1}t + Z_{2}t^{2})((1 - (Z_{1}t + Z_{2}t^{2})t_{1})))] + \kappa(Z_{1}t + Z_{2}t^{2})((1 - (Z_{1}t + Z_{2}t^{2})t_{1}))) + \kappa(Z_{1}t + Z_{2}t^{2})((1 - (Z_{1}t + Z_{2}t^{2})t_{1}))))$$
(29)

$$[Z_1t + Z_1t^2]^2 = Z_1^2t^2 + 2Z_1Z_2t^3 + Z_2^2t^4$$
(30)

$$K_{p}V'(h)t - K_{p}Z_{1}t + \omega t - K_{p}t_{2}V'(h)Z_{1}t^{2} + \frac{K_{p}}{2}V'(h)t^{2} - K_{p}Z_{2}^{2}t^{2} + K_{p}Z_{2}^{2}t_{1}t^{2} + \kappa Z_{1}t_{1}^{2} - \omega t_{2}Z_{1}t^{2} + \frac{\omega}{2}t^{2} = (K_{p}V'(h) - K_{p}Z_{1} + \omega)t + ((\kappa - K_{p}t_{2}V'(h) - \omega t_{2})Z_{1} - K_{p}Z_{2} + K_{p}t_{1}Z_{1}^{2} + \frac{1}{2}K_{p}V'(h) + \frac{\omega}{2}t^{2} = (K_{p}V'(h) - K_{p}Z_{1} + \omega)t + ((\kappa - K_{p}t_{2}V'(h) - \omega t_{2})Z_{1} - K_{p}Z_{2} + K_{p}t_{1}Z_{1}^{2} + \frac{1}{2}K_{p}V'(h) + \frac{\omega}{2}t^{2} = (K_{p}V'(h) - K_{p}Z_{1} + \omega)t + ((\kappa - K_{p}t_{2}V'(h) - \omega t_{2})Z_{1} - K_{p}Z_{2} + K_{p}t_{1}Z_{1}^{2} + \frac{1}{2}K_{p}V'(h) + \frac{\omega}{2}t^{2} = (K_{p}V'(h) - K_{p}Z_{1} + \omega)t + ((\kappa - K_{p}t_{2}V'(h) - \omega t_{2})Z_{1} - K_{p}Z_{2} + K_{p}t_{1}Z_{1}^{2} + \frac{1}{2}K_{p}V'(h) + \frac{\omega}{2}t^{2} = (K_{p}V'(h) - K_{p}Z_{1} + \omega)t + ((\kappa - K_{p}t_{2}V'(h) - \omega t_{2})Z_{1} - K_{p}Z_{2} + K_{p}t_{1}Z_{1}^{2} + \frac{1}{2}K_{p}V'(h) + \frac{\omega}{2}t^{2} + \frac{1}{2}K_{p}V'(h) +$$

$$0 = \mathbf{K}_{\mathbf{p}} \mathbf{V}'(\mathbf{h}) \mathbf{t} - \mathbf{K}_{\mathbf{p}} \mathbf{Z}_{\mathbf{l}} \mathbf{t} + \boldsymbol{\omega}$$
(31)

$$Z_{1}^{2} = (\kappa - K_{p}V'(h) - \omega t_{2})Z_{1} - K_{p}Z_{2} + K_{p}t_{1}Z_{1}^{2} + \frac{1}{2}K_{p}V'(h) + \frac{\omega}{2}$$
(32)

$$Z_{1} = V(h) + \frac{\omega}{K_{P}}$$
(33)

$$Z_{2} = \frac{1}{K_{p}} [(\kappa - \omega t_{2} - K_{p} t_{2} V'(h)) Z_{1} + (K_{p} t_{1} - 1) Z_{1}^{2} + \frac{1}{2} K_{p} V'(h) + \frac{\omega}{2}$$
(34)

$$V'(h) < \frac{1}{2A}\sqrt{4AC + B^2} - \frac{B}{2A}$$
 (35)

$$A = 1 + K_{p}(t_{2} - t_{1})$$
(36)

$$\mathbf{B} = (\kappa + \frac{K_{\rm P}}{2}) - 2\omega(t_2 - t_1 + \frac{1}{K_{\rm P}})$$
(37)

$$C = (t_2 - t_1)\frac{\omega^2}{K_p} + \omega(\frac{\kappa}{K_p} - \frac{\omega}{K_p^2} + \frac{1}{2})$$
(38)

$$V(h) < \frac{K_P}{2} - \frac{\omega}{K_P}$$

(39)

#### Results

The 8 AT was simulated using NEDC to test the model's validity by comparing the obtained fuel consumption result to the actual experimental result. The comparison of the obtained velocity speed profile with the NEDC reference speed profile is shown in figure. The vehicle model in Figure. 1 drives in accordance with the NEDC reference speed profile, as can be seen. The vehicle feedback speed shows no significant fluctuations throughout the NEDC cycle, implying that the developed vehicle model is reliable. According to the 8 AT model has a NEDC estimated fuel consumption of 6.934 L/100 km, with a 1.97% error? The difference in error is primarily due to the 4 AT gearbox model's exclusion of a lock-up gear-shifting schedule, which would have increased design complexity and used more computational resources. Furthermore, external variables such as engine working temperature, power consumption of electrical auxiliary systems, and the test driver's driving style pattern were not taken into account in the vehicle power train model. Because the error rate of 1.97% is considered acceptable, the 8 AT vehicle model was validated. Then, using the aforementioned methods, the gear ratio sets for 8 AT were created. The results of the generated gear ratio sets for 8 AT are shown in Table. The first and top gear ratios are similar in both design methods, but the intermediate gear ratios are different. From lower to higher gear, the progressive method has a decreasing ratio spread. This means that the lower gear has a larger speed difference v and the higher gear has a smaller speed difference v, resulting in better drivability, acceleration performance, and fuel economy. There are five sections to this paper. The microscopic model of traffic flow is detailed in category 1. The part two present a new microscopic model that considers vehicle interaction. The stability of the car following model is investigated in Category 3. Section 4 is where numerical simulations and analyses of experimental results are carried out. The eight speed transmission model consumes 1.8% less fuel than six speed transmission model based on the eco mode pattern, indicating that the proposed design method is feasible and effective.

Table 1: Simulation results comparison of the fuel consumption rate between the actual experimental result and simulated result of 2022

Result type	Fuel consumption rate	Error Percentage
Actual experimental results	4.7 L / 100 km	-
Simulation results	6.27 L / 100 km	1.28%

The geometric progression method maintains a consistent ratio spread across all gears, resulting in improved driving comfort and a smoother shift process. Both gear ratio sets were tested to see if they adhered to the Lepelletier gear set mechanism's design configuration and limit. The evaluation results for both gear ratio set design methods are shown in Table 4. According to Tables 1,2,3 and 4 the gear ratio set generated using the geometric progression method does not meet the design configuration and limit of the Lepelletier gear set mechanism because g1 is less than 1 and g3 is less than g2. As a result, the progressive method gear ratio set is chosen because it meets the design requirement and limitation of the Lepelletier gear set mechanism. The driving condition and power-speed diagrams of the selected gear ratio set for the 8 AT at maximum throttle were then constructed, as shown in Figs. 2 and 3, respectively. The dashed-black lines in these figures represent road load resistance and road load power at various road gradients. The first gear ratio is capable of providing sufficient traction force greater than the initial design grade ability assumption of 15%, with adequate reserve traction force in the event that extra vehicle load is applied. All models had been simulated based on motor vehicle emissions groups to investigate fuel consumption and acceleration effectiveness.

rent methods

Geometr	ic progression m	ethod	Pro	ogressive method	
Gear number	Gear ratio	Ratio spread	Gear number	Gear ratio	Ratio spread
<i>i</i> 1	3.1	-	<i>i</i> 1	4.2	-
i2	2.3	1.4	i2	2.5	1.26
i3	2.1	1.4	i3	1.2	1.24
<i>i</i> 4	1.1	1.4	<i>i</i> 4	1.1	1.21
i5	0.92	1.4	<i>i</i> 5	0.84	1.19
<i>i</i> 6	0.52	1.3	<i>i</i> 6	0.6	1.15



Fig 2: Power-speed diagram

Using the power-speed diagram shown in, the best acceleration performance up shift points was determined.



Fig 3: Sensitivity Coefficient diagram

Table 3: Overall simulation result for two different methods designed gear ratio sets for 8 AT based on two different methods

Gear ratio set	<i>g</i> 1	<i>g</i> 2	<i>g</i> 3
Geometric progression method	0.92	1.93	2.01
Progressive method	1.309	1.93	2.01

Table 4: Overall simulation result	Ilt for all gear-shifting patterns
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Type of shifting pattern	Fuel consumption rate	0 – 100 km/h acceleration	Error in vehicle speed with reference speed
Best acceleration performance	6.03 L/70 km	10 s	0.28%
Best fuel economy	4.47L/800 km	12 s	0.42%
Combination	2.28 L/90 km	12 s	0.76%

#### Conclusion

The gear ratio set and gear shift pattern for a eight speed Lepelletier automatic transmission are proposed in this paper. According to simulation results, a similar vehicle with a higher number of gear ratios has better fuel economy than a vehicle with a lower number of gear ratios. The simulation results also show that the vehicle's acceleration, fuel economy, and shifting experience are all affected by the gear shifting pattern. More importantly, it has been demonstrated that studying vehicle performance using computer simulation technology can be done effectively and efficiently, provided that the model is correct, accurate, and validated. Simulation software technology is an effective method for assisting the automotive industry during the research and development stages, as it reduces the required development time, cost, effort, and resources when compared to the traditional method. The vehicle power-train model developed in this study can be used to quickly assess the fuel

consumption performance of any internal combustion engine vehicle by simply changing the vehicle parameters to the corresponding data. As a result, the design method proposed in this article for determining a vehicle transmission's gear ratio set and gear shifting pattern is feasible and effective, and it can serve as a great starting point for future research on optimizing gear shift operations in automatic transmissions.

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