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Dr. Akhilesh Kumar
Assistant Professor, Department of Information Technology, Gaya College, Gaya Bihar, India

Corresponding Author:
Dr. Akhilesh Kumar
Assistant Professor, Department of Information Technology, Gaya College, Gaya Bihar, India

Analyzing efficiency of decision-making units in data envelopment analysis

Dr. Akhilesh Kumar

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Abstract
A variety of irrigation management systems are evaluated using Data Envelopment Analysis (DEA) to see which one is the most cost-effective, environmentally friendly, and socially responsible. Accordingly, an irrigation management project report serves as the basis for the comparison of different ranks in MCDM and DEA. There are DEA approaches termed CCR, BCC, and RCCR that use criterion weights integrated with the inclusion of assurance areas to increase the analysis' discriminating power and reach the ranking of strategies. The findings show that DEA is a good alternative or supplement to MCDM, and including management preferences into the DEA methodology yields comparable outcomes to MCDM procedures.

Keywords: Data envelopment analysis, MCDM, irrigation management

1. Introduction
Competitive marketplaces nowadays have alerted businesses to the need of improving their performance in order to better compete against their rivals. In order to get the most out of your time and resources, you must take into account a variety of aspects. You may use a variety of methods to evaluate efficiency. A non-parametric approach to efficiency measurement is DEA. A wide range of fields, including management science, operational research and system engineering as well as decision analysis and so on are now able to employ DEA as a tool. Charnes *et al*. (1978) [16] is one of the most frequently cited works in this field. Charnes, Cooper, and Rhodes (CCR) stated that the greatest ratio of weighted outputs to weighted inputs may be used to measure the efficiency of a DMU. This is only possible if all DMUs have a ratio of less than or equal to one in mind. To increase the efficiency and overcome the flaws of prior models, a number of scientists set out to develop new ideas.

2. Literature Review
Rafael Benítez, Vicente Coll-Serrano, and Vicente J. Bolós (2021) [1]. An interactive web application (deaR-shiny) that uses data envelopment analysis to assess efficiency and productivity is described in this research (DEA). Online DEA software is presently lacking, and deaR-shiny intends to address that need by providing practitioners and scholars alike with free access to a broad range of DEA models (both conventional and fuzzy models). By re-creating the main results of Carlucci, Cirà, and Coccorese in 2018, who investigate the efficiency and economic sustainability of Italian regional airports using two conventional DEA models, and Kao and Liu in their papers published in 2000 and 2003, who calculate the efficiency scores of university campuses, we demonstrate how to use the web app.

Nafiseh Javaherian, Ali Hamzehee, and Hossein Sayyadi Tooranloo (2020) [2]. For ranking and comparison purposes and to distinguish between efficient and wasteful units, data envelopment analysis (DEA) is a strong technique. In the actual world, inputs and outputs are often imprecise or nondeterministic, rendering classic DEA models unsuitable for issues involving numerous steps of decision-making with intermediate results. For decision-making units with two-stage architectures and triangular intuitionistic fuzzy data, this research provides a novel DEA model. Two-stage DEA models are initially introduced in this work. Then, the study discusses how intuitionistic fuzzy coefficients may be used to modify these models, and lastly how arithmetic operators for intuitionistic fuzzy numbers can be utilized for a two-stage structure conversion. An exemplary numerical example is presented to demonstrate how the suggested technique works.

Bao Jiang, Shuang Feng, Jinwu Gao, And Jian Li (2020) [3] Decision-making units (DMUs) need to evaluate efficiency in terms of returns to scale (RTS) in order to allocate resources and make scientific decisions, but this form of assessment becomes problematic when the DMUs are operating in a random environment. Uncertain random data envelopment analysis is explored in this study to answer for the fact that DMU input and output variables are undeterministic random variables. These uncertain random variables are handled by chance theory and two evaluation models for rising returns to scale (IRS) and declining returns to scale (DRS) are presented, respectively. We present a numerical example to demonstrate the evaluation outcomes of these models in addition to turning the two uncertain random models into similar forms.

Robert Stefko, Beata Gavurova & Kristina Kocisova (2018) [4] Data Envelopment Analysis (DEA) factors such as the utilization of medical technologies (MR, CT) are examined in this article to see how they affect the outcomes of the assessment of healthcare facilities' efficiency and appropriateness. All areas' projected efficiencies were shown to be directly linked to changing values of variables across time, according to an examination of the data obtained. Over time, the locations with the lowest values of the variables obtained the highest levels of efficiency. A fascinating finding was that the addition of factors such as the number of MR, CT and medical equipment combined did not have a significant influence on the overall projected efficiency of healthcare facilities over time.

Xiao-Li Meng & Fu-Gui Shi (2017) [5] Data envelopment analysis (DEE) may be used to assess the efficiency of decision-making units based on historical input and output data. In all, the effort contributes three times as much. A time series approach is used to analyze and forecast data since the input and output of the decision-making unit being examined change over time. Another factor to consider is that there are numerous sample decision-making units, each of which is subdivided into many sample standards in terms of production strategy, and the constraint condition is one of these standards. Using this information, it is possible to see how the assessed decision-making unit compares to a sample of constraint-constrained decision-making units. A binary search tree approach is used in the model to choose the sample standard that is most similar to an assessed decision-making unit's behaviour. To demonstrate the suggested concept, we provide two numerical examples.

3. Data envelopment analysis

3.1 Theory

Based on the various inputs and outputs of a homogeneous collection of DMUs, DEA is a productivity analysis model. Multi-input, multi-output efficiency is measured by the efficiency score:

$$\text{Efficiency} = \frac{\text{weighted sum of outputs}}{\text{weighted sum of inputs}}$$

Charnes *et al*. established a model known as CCR, in which the relative efficiency score of a test DMU p is calculated by solving the following equation.

$$\max \quad \frac{\sum_{k=1}^{s} u_k y_{kp}}{\sum_{j=1}^{m} v_j x_{jp}}$$

$$\text{s.t.} \quad \frac{\sum_{k=1}^{s} u_k y_{ki}}{\sum_{j=1}^{m} v_j x_{ji}} \le 1 \quad i=1,2,\ldots,n \quad ; \quad u_k, v_j \ge 0 \quad k=1,2,.., s \quad j=1,2,.., m$$

It is important to note that the DMU's output (yki), the quantity of input (xji) used, and the weight assigned to each of these factors are shown in the table below. The fractional equation above may be transformed into a linear programming issue as follows.

$$\max \sum_{k=1}^{s} u_k y_{kp}$$
$$\text{s.t.} \sum_{j=1}^{m} v_j x_{jp} = 1 \ ; \ \sum_{k=1}^{s} u_k y_{ki} - \sum_{j=1}^{m} v_j x_{ji} \le 0 \quad i=1,2,..,n \ ; \ u_k, v_j \ge 0 \quad k=1,2,.., s \quad j=1,2,.., m \quad (3)$$

The relative efficiency of all DMUs is determined by solving Equation 3 n times. DMUs combine to form an efficient frontier by selecting input and output weights that optimize their efficiency scores. If a DMU's score is 1 or above, it's regarded efficient; if it's less than 1, it's deemed inefficient. The Banker Charnes Cooper (BCC) model is another fundamental DEA model that varies from the CCR model in terms of scale assumptions. Envelopment surfaces may be constant-return to scale (CRS) or variable-return to scale (VRS) as assessed in the CCR model and BCC model, respectively, as illustrated in Figure 1.

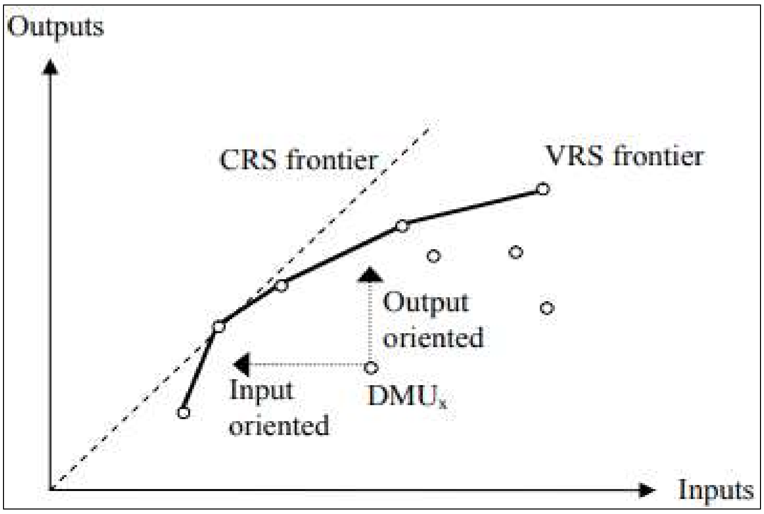

Fig 1: Envelopment Surfaces and Orientation

The CRS envelopment surface models presume that an increase in inputs will lead to an equal rise in outputs. It is, nevertheless, very uncommon for markets to use CRS surfaces for manufacturing. For example, the DEA model for variable returns to scale (VRS) was designed to accommodate non-proportionality and to represent the reality that the production process may display growing, stable and declining returns to scale. In other words, the VRS model allows for a non-proportional rise in output levels when input values are increased. CRS surface approaches the VRS boundary by linking outermost DMUs, including that one. There is further evidence that DMUs are more efficient than CCR in the BCC paradigm. Andersen and Petersen devised a novel strategy to improve the ranking abilities of DEA, which is a shortened version of the CCR model. Super efficiency, RCCR, or reduced form of CCR are all terms used to refer to the RCCR concept. There are no constraints on DMUs in this model, hence an efficiency score of more than 1 may be achieved by removing the test DMU p from the constraint list.

Input-oriented and output-oriented DEA models have been created to quantify efficiency in two ways. For a DMU that can provide as much output as possible with the available resources, output-oriented models are used, whereas input-oriented models are used for DMUs that can produce as much output as possible while utilizing the least number of resources (Figure 1). Efficiencies based on input are scored from zero to one, while efficiencies based on output are scored from one to infinite. In both circumstances, a score of 1 is the most effective. This study uses an input-oriented model because it focuses on water resources sustainability, which is congruent with the research topic of this study.

3.2 Weighting and ranking in DEA

The DEA models should take into account two critical elements. There is no restriction on the weights that may be used to calculate DMU efficiency ratings under all DEA models. To attain high efficiency, units might indulge in incorrect weights for input and output. Relative relevance levels of multiple inputs and outputs may be integrated using weight constraints. According to Thompson *et al*, the use of assurance regions (AR) for weight limits is more frequent and reflects marginal rates of replacement in the literature. Defining upper and lower limits for each input and output weight is part of configuring AR. It is possible to discover the lower and upper boundaries of each weight by lowering and raising the weight by a certain percentage (for example, 10%). (a) "Do you believe that the relevance of input measure in assessing DMUs might be as low [or as high] as z percent?"; or (b) "Should, as a matter of policy, the importance of input measure I in the evaluation of DMUs be permitted to be as low (or high).?". The DM preference range on input weights (Equation 4) may be introduced to the linear programming issue after the upper and lower limits of all inputs have been established. This results in more dependable and reasonable outcomes.

$$v_i \geq \frac{\alpha_i}{\beta_j} v_j \ , \ v_i \leq \frac{\beta_i}{\alpha_j} v_j$$

Second, the classic DEA models do not allow for ranking DMUs, particularly the most efficient ones, in terms of efficiency. Cross-efficiency ranking method, super-efficiency ranking method, and benchmarking ranking method have all been developed in an effort to better distinguish between different scores.

3.3 Application of DEA

These models are tested both with and without extra weight limitations on the same set of data. Defining upper and lower boundaries for every criterion is necessary in order to include these limitations. Assuming the lower and upper dispersion are both equal and) boundaries by increasing/decreasing by 10% is reasonable since the stated criterion weights are single scalar values. At the high end of the scale, for example, lower is 0.10 for the first minimizing criterion (initial cost). When the percentage of this value is 10%, the lower and upper limits are determined to be 0.09 and 0.11, respectively. In this example, all criteria are considered to have a same percentage of dispersion on their boundaries for a given amount of dispersion, and if these bounds are accurately defined by the DMs, these ranges may be employed.

Table 1: The Data Used for DEA

Alternative	CR1 {I}	CR2 {I}	CR3 {I}	CR4 {I}	CR5 {O}	CR6 {O}	CR7 {O}
A1	0.556	0.500	0.625	0.714	2.800	1.800	2.800
A2	0.455	0.500	0.625	0.556	3.000	1.800	3.200
A3	0.500	0.556	0.714	0.714	2.600	1.600	3.000
A4	0.417	0.556	0.714	0.556	2.800	1.600	3.400
A5	0.556	0.500	0.625	0.714	2.600	1.800	2.200
A6	0.455	0.500	0.625	0.556	2.800	1.800	2.600
A7	0.500	0.556	0.714	0.714	2.400	1.600	2.400
A8	0.417	0.556	0.714	0.556	2.600	1.600	2.800
A9	0.556	0.625	0.556	0.714	2.400	2.000	2.000
A10	0.455	0.625	0.556	0.556	2.600	2.000	2.400
A11	0.500	0.714	0.625	0.714	2.200	1.800	2.200
A12	0.417	0.714	0.625	0.556	2.400	1.800	2.600
A13	0.714	0.500	0.556	0.625	3.000	2.000	2.600
A14	0.556	0.500	0.556	0.500	3.200	2.000	3.000
A15	0.625	0.556	0.625	0.625	2.800	1.800	2.800
A16	0.500	0.556	0.625	0.500	3.000	1.800	3.200
A17	0.714	0.500	0.556	0.625	2.800	2.000	2.000
A18	0.556	0.500	0.556	0.500	3.000	2.000	2.400
A19	0.625	0.556	0.625	0.625	2.600	1.800	2.200
A20	0.500	0.556	0.625	0.500	2.800	1.800	2.600
A21	0.714	0.625	0.500	0.625	2.600	2.200	1.800
A22	0.556	0.625	0.500	0.500	2.800	2.200	2.200
A23	0.625	0.714	0.556	0.625	2.400	2.000	2.000
A24	0.500	0.714	0.556	0.500	2.600	2.000	2.400
A25	0.714	0.455	0.500	0.556	3.200	2.200	2.600
A26	0.556	0.455	0.500	0.455	3.400	2.200	3.000
A27	0.625	0.500	0.556	0.556	3.000	2.000	2.800
A28	0.500	0.500	0.556	0.455	3.200	2.000	3.200
A29	0.714	0.455	0.500	0.556	3.000	2.200	2.000
A30	0.556	0.455	0.500	0.455	3.200	2.200	2.400
A31	0.625	0.500	0.556	0.556	2.800	2.000	2.200
A32	0.500	0.500	0.556	0.455	3.000	2.000	2.600
A33	0.714	0.556	0.455	0.556	2.800	2.400	1.800
A34	0.556	0.556	0.455	0.455	3.000	2.400	2.200
A35	0.625	0.625	0.500	0.556	2.600	2.200	2.000
A36	0.500	0.625	0.500	0.455	2.800	2.200	2.400

4. Results

Table 2 summarizes the efficiency scores of all the DEA models, with the values in red denoting efficiency levels greater than or equal to 1. Because 12 and 15 of the 36 possibilities received scores of 1 in the CCR and BCC models, the findings of the CCR and RCCR models are indistinguishable. The RCCR efficiency, on the other hand, seems to be rather different. RCCR model is not adequate for a complete ranking of options, since CCR and BCC alone are not a good discriminator. However, it can be argued that DEA may be improved by including weight limits as a means of determining the preferences of the decision makers as well as enhancing the discriminating power of DEA. Alternative 26 is the only effective DMU in all models, as can be observed.

Table 2: The Efficiency Scores Determined

Alternatives	CCR	BCC	RCCR	CCR/w	BCC/w	RCCR/w
A1	0.8577	0.9394	0.8577	0.7453	0.8565	0.7453
A2	1.0000	1.0000	1.0162	0.9045	0.9466	0.9045
A3	0.8459	0.9052	0.8459	0.7117	0.8254	0.7117
A4	1.0000	1.0000	1.1591	0.8552	0.9024	0.8552
A5	0.7927	0.9394	0.7927	0.6509	0.8565	0.6509
A6	0.9740	1.0000	0.9740	0.7966	0.9466	0.7966
A7	0.7796	0.9052	0.7796	0.6176	0.8254	0.6176
A8	0.9601	1.0000	0.9601	0.7523	0.9024	0.7523
A9	0.8257	0.9154	0.8257	0.5987	0.8365	0.5987
A10	1.0000	1.0000	1.0080	0.7329	0.9212	0.7329
A11	0.8196	0.9009	0.8196	0.5628	0.8013	0.5628
A12	1.0000	1.0000	1.0197	0.6867	0.8737	0.6867
A13	0.8264	0.9091	0.8264	0.7378	0.8384	0.7378
A14	0.9226	0.9508	0.9226	0.9027	0.9369	0.9027
A15	0.7724	0.8472	0.7724	0.7152	0.8219	0.7152

A16	0.9444	0.9589	0.9444	0.8709	0.9114	0.8709
A17	0.8264	0.9091	0.8264	0.6542	0.8384	0.6542
A18	0.8915	0.9508	0.8915	0.8036	0.9369	0.8036
A19	0.7058	0.8472	0.7058	0.6246	0.8219	0.6246
A20	0.8867	0.9589	0.8867	0.7670	0.9114	0.7670
A21	0.8333	0.9091	0.8333	0.5986	0.8145	0.5986
A22	0.9130	0.9524	0.9130	0.7399	0.9107	0.7399
A23	0.7384	0.8521	0.7384	0.5702	0.7967	0.5702
A24	0.9192	0.9589	0.9192	0.6973	0.8764	0.6973
A25	1.0000	1.0000	1.0000	0.8312	0.9009	0.8312
A26	1.0000	1.0000	1.1178	1.0000	1.0000	1.0554
A27	0.8485	0.9091	0.8485	0.8046	0.8873	0.8046
A28	1.0000	1.0000	1.0847	0.9757	0.9834	0.9757
A29	1.0000	1.0000	1.0000	0.7413	0.9009	0.7413
A30	1.0000	1.0000	1.0000	0.9002	1.0000	0.9002
A31	0.8264	0.9091	0.8264	0.7122	0.8873	0.7122
A32	0.9788	1.0000	0.9788	0.8654	0.9834	0.8654
A33	1.0000	1.0000	1.0000	0.6822	0.8773	0.6822
A34	1.0000	1.0000	1.1341	0.8367	0.9786	0.8367
A35	0.8333	0.9091	0.8333	0.6548	0.8646	0.6548
A36	1.0000	1.0000	1.0295	0.7967	0.9530	0.7967

Table 3 shows the results of MCDM approaches and weighted DEA models, which show the ranking pattern of alternatives. Each approach has a somewhat different rank order. In both the MCDM and the DEA models, alternative 26 is the best option to pursue. However, under the ELECTRE-4 approach, options 26, 28 are tied for first place, while alternatives 2, 30, and 32 are placed sequentially in Table 3. In ELECTRE-3, ELECTRE-4, CCR/w, and RCCR/w, the first three rankings are the same, whereas CCR/w and RCCR/w ranks are identical. Alternative 23 and alternative 11 are likewise the least favoured options, according to the MCDM and DEA models. According to the assessments, the MCDM techniques and weighted DEA models meet the requirements of the DM(s) to identify the best and worst decision.

Table 3: Increasing Rank Order of Alternatives

Rank		**MCDM Techniques**				**DEA Models**		
	EL.-3	EL.-4	CP (p=1)	CP (p=2)	CP (p=∞)	CCR/w	BCC/w	RCCR/w
1	26	26	26	26	28	26	26	26
2	28	28	28	28	26	28	30	28
3	2	2	14	14	14	2	28	2
4	4	30	30	2	2	14	32	14
5	14	32	2	16	16	30	34	30
6	16	14	25	25	27	16	36	16
7	25	34	16	27	25	32	2	32
8	30	4	32	30	32	4	6	4
9	32	36	34	32	13	34	14	34
10	27	10	4	13	1	25	18	25
11	34	16	27	18	15	27	10	27
12	36	18	18	4	4	18	16	18
13	3	12	36	1	30	36	20	36
14	8	25	13	6	18	6	22	6
15	18	27	6	34	6	20	8	20
16	33	6	29	15	20	8	4	8
17	1	8	20	20	36	1	29	1
18	13	20	1	36	34	29	25	29
19	6	22	22	22	22	22	31	22
20	29	29	15	3	31	13	27	13
21	15	24	8	29	3	10	33	10
22	20	31	31	31	8	15	24	15
23	22	33	10	8	10	31	12	31
24	10	1	33	10	24	3	35	3
25	24	35	3	24	29	24	5	24
26	31	3	24	17	5	12	1	12
27	12	13	17	5	19	33	17	33
28	35	15	12	19	17	35	13	35
29	17	5	35	33	35	17	9	17
30	5	17	5	12	12	5	7	5
31	7	19	19	35	33	19	3	19
32	21	21	21	7	7	7	15	7

33	19	7	7	21	21	9	19	9
34	9	9	9	9	9	21	21	21
35	11	11	23	23	23	23	11	23
36	23	23	11	11	11	11	23	11

Understanding the degree to which the rankings of various methodologies are associated may be accomplished by calculating the Spearman rank correlation coefficient (r). No correlation, no connection, and total disagreement are all represented by Spearman correlation r values of 1, 0, and -1; accordingly. Correlation data is included in Table 4 for each model in Table 3 and Table 4. When compared to other weighted DEA models, there are significant relationships between the various MCDM approaches. As a consequence, the introduction of extra restrictions to include value judgments into DEA seems to yield results that are associated with certain MCDM techniques. For the current irrigation policy making issue, DEA models, particularly the RCCR model, applied with weight limits achieved through realistic lower and upper bounds, are appropriate.

5. Conclusion

Irrigation policies may be prioritized using the DEA method, which can be used to rank alternative policies. Project report findings using MCDM approaches, such as CCR, BCC and RCR, are compared with the outcomes of the DEA models in the CCR, BCC and RCR datasets. It is not possible to rank DMUs and include their preference judgements into the analysis using the usual approaches, thus extra weight restrictions are added to DEA models and well-correlated findings may be achieved. Although there is no widely acknowledged way for comparing DEA and other MCDM tools, integrating DM preferences does considerably improve the correlation between DEA and outranking/distance-based approaches, therefore employing both methods together will boost the credibility of the judgments. DEA models with fairly tight constraints on the weights of criteria are used to get this conclusion (10 percent). For future investigations, tighter or looser constraints in the DEA may be used, or other kinds of MCDM approaches can be used in place of DEA.

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